CALCULATING PROBABILITIES

Assuming equally likely outcomes, the probability of an event can be calculated as

\[ p(\text{event}) = \frac{\text{number of outcomes in event}}{\text{total number of outcomes}} \]

**Example:** The following table summarizes the numbers of defective and nondefective medical devices produced by two plants:

<table>
<thead>
<tr>
<th></th>
<th>Plant A</th>
<th>Plant B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Defective</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Nondefective</td>
<td>180</td>
<td>70</td>
</tr>
</tbody>
</table>

a) The probability of selecting a defective device is

\[ p(\text{defective}) = \frac{\text{Number of defective devices}}{\text{Total number of devices}} = \frac{20 + 30}{20 + 30 + 180 + 70} = \frac{50}{300} = 0.17 \]

b) The probability of selecting a device produced by plant B is

\[ p(\text{plant B}) = \frac{\text{Number of devices from plant B}}{\text{Total number of devices}} = \frac{30 + 70}{300} = \frac{100}{300} = 0.33 \]

c) The probability of selecting a defective device from plant B

\[ p(\text{defective} \cap \text{plant B}) = \frac{\text{Number of defective devices that are from plant B}}{\text{Total number of devices}} = \frac{30}{300} = 0.1 \]

**Some useful Probability Properties**

The probability of an event A not occurring is given by

\[ \text{NOT} A \quad P (A') = 1 - P (A) \]

The probability of either event A or event B occurring is given by

\[ A \text{ or } B \quad P (A \cup B) = P (A) + P (B) - P (A \cap B) \]

If the events A and B are mutually exclusive (cannot both occur at the same time), then

\[ P (A \cap B) = 0 \quad \text{and} \quad P (A \cup B) = P (A) + P (B) \]

**Remember:**

The sum of the probabilities of all possible outcomes is 1

The probability of each outcome is a number between 0 and 1

**Example:** Being married and being over 30 years old are not mutually exclusive events.

Being a teenager and being over 30 years old are mutually exclusive events.
The conditional probability of event A, given that event B has occurred is

\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]

or rearranging \( P(A \cap B) = P(A \mid B) \times P(B) \)

If two events A and B are independent (the occurrence of one event does not influence the probability of the other), then

\[ P(A \mid B) = P(A) \quad \text{and} \quad P(A \cap B) = P(A) \times P(B) \]

Example: Consider the table below with the probabilities as shown:

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colourblind</td>
<td>0.04</td>
<td>0.002</td>
</tr>
<tr>
<td>Not colourblind</td>
<td>0.47</td>
<td>0.488</td>
</tr>
</tbody>
</table>

Find:

a) The probability of selecting a colourblind male

\[ P(\text{Male} \cap \text{colorblind}) = 0.04 \quad (\text{Taken directly from the table}) \]

b) The probability of selecting a non colourblind female

\[ P(\text{Not colourblind} \cap \text{female}) = 0.488 \quad (\text{Taken directly from the table}) \]

c) The probability of selecting a male

\[ 0.04 + 0.47 = 0.51 \]

Explanation: A male is colourblind or not colourblind and since events ‘colourblind’ and ‘not colourblind’ are mutually exclusive, we can use

\[ P(A \cup B) = P(A) + P(B) \]

d) The probability of selecting a colourblind person

\[ 0.04 + 0.002 = 0.042 \]

e) The probability of selecting either a male or a colourblind person

\[ P(\text{male} \cup \text{colourblind}) = P(\text{male}) + P(\text{colourblind}) - P(\text{male} \cap \text{colourblind}) \]

\[ = 0.51 + 0.042 - 0.04 \]

\[ = 0.512 \]

e) The probability of selecting either a male or a colourblind person

\[ P(\text{male} \cup \text{colourblind}) = P(\text{male}) + P(\text{colourblind}) - P(\text{male} \cap \text{colourblind}) \]

\[ = 0.51 + 0.042 - 0.04 \]

\[ = 0.512 \]

f) The probability of being colourblind, if the person is male:

\[ P(\text{colourblind} \mid \text{male}) = \frac{P(\text{colourblind} \cap \text{male})}{P(\text{male})} \]

\[ = \frac{0.04}{0.51} \]

\[ = 0.078 \]
g) Determine if two events ‘being a male’ and ‘being colourblind’ are independent.

If two events are **independent**, then

\[ P(A \cap B) = P(A)P(B) \]

\[
P(\text{colourblind} \cap \text{male}) = 0.04
\]

\[
P(\text{colourblind}) \cdot P(\text{male}) = (0.042)(0.51) = 0.02
\]

Not equal; therefore, dependent.